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IMPLEMENTATION OF SINGULAR VALUE DECOMPOSITION FOR ADDING WATERMARK ON TEXT DOCUMENTS

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Abstract

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Watermark is a part of steganography that is used by many users in electronic and non-electronic documents. Watermark certainly does not guarantee the safe use or modification of the document, but only the legitimacy of ownership of the document. Watermarking is a part of steganography that is used by many users in electronic and non-electronic documents. Watermarking certainly does not guarantee the safe use or modification of the document, but only the legitimacy of ownership of the document. Establishing a digital watermark will improve the owner's document security. Hiding a label by inserting datum bits in the image segment using Singular Value Decomposition (SVD), as a part of Watermarking. Watermark domains are digital documents in the form of videos, images, text, sound. Watermarking applications built by applying singular value decomposition to numbers

Keywords: Singular_Value_Decompostion, image, document, robustness, watermark

1. Introduction

Watermarking is a derivative of steganography, which has resistance to attacks (hackers), even though the inserted data is visible in the document that using by other parties. A watermark (watermark) is data that attached to a digital document, whether video, audio, text, where its presence does not affect the further processing of the document[2][8][9]. This watermarking program works by inserting a watermark image into the original image to get an image containing a watermark. Then extract it again to get the watermark image back [3]. Perhaps for other users the insertion or the attached document is just an eyesore or it may give the impression that the document or stolen from another party. There are two forms of invisible digital watermarking, namely spatial domain and frequency domain. The frequency domain technique will use image transformation in the frequency domain, among which are the discrete cosine transformation (DCT) and discrete wavelet transformation (DWT) and others. Another type of watermark embedding that is based on spatial-domain has become very popular[10].

Based on the extraction process (decoding), it is divided into 2 parts, namely: non-blind watermarking and blind watermarking. The watermark decoding (extraction) algorithm requires the original host image to be called non-blind watermarking, otherwise it is called blind watermarking. Some other purposes of watermarking can be such as Tamper-Proofing: used as a tool to identify or show that digital data has changed from the original and Feature location: only used as a description of the digital data itself. [7]

Singular Value Decomposition (SVD) is a robust and reliable orthogonal matrix decomposition method. SVD is a technique of decomposing any sized matrix to simplify data processing. The result of SVD is a singular value stored in a diagonal matrix[3][11]. SVD is a technique on the basis of numerical computation by diagonalizing matrices.

Suppose there is a matrix A where the eigenvalues of the A^TA is λ_i for each $1 \le i \le n$ where n is the number of eigenvalues, then the singular value of matrix A is : $\tau_i = \sqrt{\lambda i}$ and v_i is an eigenvector matrix A^TA.

SVD has a lot of practical and theoretical value so, it can be used for many real (m,n) matrices. For example, there is a matrix A with m rows and n columns, and the rank r≤n≤m.. Matrix A can be decomposed into 3 matrices, that is, A=USVT. The analogy of the number of rows and columns of the U, S, and VT matrices [2]. Matrix U is an orthogonal matrix mxm : $U=[u_1,u_2,...u_{r+1},...,u_m]$; Vector u_i, for i = 1,2,...,m comes from the orthogonal set $u_i^T u_j = \delta_{ij} = \{^{1,2,...,i=j}_{1,2,...,i\neq j}$ The V matrix is an nxn orthogonal matrix : $V = [v_1,v_2 ,...v_r +1,...,v_m]$; Column vector v_i, for i = 1,2,...,m from the orthogonal set : $v_i^T v_j = \delta_{ij} = \{^{1,2,..,i=j}_{1,2,..,i\neq j}$; S is an mxn diagonal matrix with a singular value (SV) on the diagonal. For i = 1, 2, ..., n, r called the singular value (SV) of matrix A, it can be proved that $\sigma_1 \ge \sigma_2 ... \ge \sigma_r \ge 0$, and $\sigma_{r+1} = \sigma_{r+2} ... = \sigma_N = 0$. For i = 1, 2, ..., n, r is called the singular value (SV) of matrix A. Column vi's is called right singular vector and column ui's is called left singular vector.

As the use of watermarks is only considered an art in document writing, but in quotes from several journals, there are many other opinions stating, among others Watermark is a technique of hiding data or information into other data to be "boarded" (host data), but people are not aware of the presence of additional data in the data host[1]. Applying Singular Value Decomposition (SVD) on the host image to determine the modified singular value by adding a watermark, then SVD is reused in the resulting matrix to calculate the modified singular value[2]. If a document that on the internet is in the form of text or images, if there is no copyright for the digital work, it will open up opportunities for copyright infringement of other people's works[3]. Based on the description above that watermarking is not only an art but part of security. The security of the document in question is not how strong the security is, but if the watermark visible, it will allow users not to think about transferring copyright from the original document maker.

2. Method

2.1 Early Preparation

The steps of the formulation of data collection in the study are :

1. Collecting image input data with various types and resolutions according to the capabilities of the application used.
2. Converting a color image to grayscale: To convert a color image that has a matrix value of r, g and b into a grayscale image with a value of s, the conversion can be done by taking the average of the values of r, g and b.
3. Collecting a number of libraries related to image processing with Singular Value Decomposition (SVD).
4. Starting from identifying image specifications, segmenting images, and applying watermarks, analyze and apply these two methods step by step.
5. Set up a database and data dictionary to connect to the application.
6. Preparation and compilation phase report.
7. Implementing the application design.
8. Perform data testing and draw conclusions.
9. Make a final report.

This section is composed of sub-sections such as data collection stage, data source/research location or research location, research time span, and problem solving analysis methods.

2.2 Processing Stage

The steps in this stage are :

1. Filtering: This stage is to prevent the emergence of edge detection that is not in line with expectations caused by the presence of noise in the image with a Gaussian Filter.
2. Grayscale: Converts RGB image pixels to grayscale. There are 2 levels of grayscale, namely: 1) from 0-127 for low grayscale and 2) from 128-255 for high grayscale. Grayscale image serves to simplify the image model, where the color image has 3 layers, namely: 1) R-layer, 2) G-layer, 3) B-layer. These three points become a reference for further processing. If the calculation process uses the RGB layer, all three are carried out in the same way, the basis because the calculation is the same, the image processing in this case is changed to grayscale. In grayscale, no longer takes into account the color remains only the degree of gray. Converting RGB to grayscale is the sum of the values of R, G, B divided by 3
3. Finding Direction, Gradient Distance, Partition: the intensity of the gray level will change drastically so that the location will be obtained by determining the gradient of an image. The gradient at each pixel of the smoothed image is determined by the Robert operator. The first step is to estimate the gradient according to the number of direction operators by applying the kernel. The image is partitioned into several blocks where each block has a size of the kernel matrix, the purpose of partitioning the image into several blocks is to segment several blocks where each block size can be adjusted.
4. Image Block Operation: Operation of each pixel in the block for all existing blocks, so that each block performs the same operation. The operation in question is to find the neighboring points of the pixel coordinates (x,y).
5. Determination of Target: After the neighboring points of all coordinates in the block are obtained, the next step is to find the difference in the gray level values for each, where d is the direction, r and l are neighbors of a coordinate and mn is the location of a block. There are four different gray level values according to d or the direction used.
6. Watermark Algorithm Design: After the identification stage above is carried out, the next step is to design the algorithm that will be used for marking research document files.
7. User Interface Design system, the design of the forms required by the system in order to obtain output according to user needs.
8. Test data collection, the test data collected is an image document file in the form of softcopy

3. Result And Discussion

3.1 Basic Concepts of Applying Watermark

In this study, a watermark application design scheme using the Single Value Decomposition (SVD) formula to identify the document, namely the insertion (Encoding) of the watermark in the document and extraction (Decoding).

1. Embedding (Insertion). Insert the watermark into the carrier medium that will add the watermark to the digital document. This section uses 4 variables (W, I, K, IW) and each variable represents each component in the document watermarking process. Where I is a watermark, W is the watermark to be inserted, K is the key media. E is dependent on the insertion of W, carrier and lock as a form of safety. This function will generate Iw, that is, the media with the watermark inserted. The SVD function with 3 matrices S, U, V. The S matrix by inserting a watermark multiplied by a constant value as the intensity value in the range [(0,1) ... (1)] with a distance (0,1). The process is continued by composing the modified S matrix and then combining it with the modified S matrix before combining it with the U and V matrices of the original image. This formulation satisfies $A_W=US_WV^T$.
2. Decoding. Decoding to retrieve the watermark on the document media. There are variables Iw and W where Iw is the document media with a watermark, while W is the extracted watermark where these variables are different from the variables in the insertion function because there is a possibility $\hat{I}_W$ that previous modifications, causing the results of the

watermark that there may be significant differences. .If you want to take the watermark back, it's just a form of verification.

3. The singular value decomposition algorithm is as follows: input matrix A, output the orthogonal matrix U, V and singular matrix S, so that $A=USV^T$
   1) For each A^TA matrix that forms an eigenvalue λ_i for each $1 \le i \le n$ the singular value (δ_i) of matrix A is calculated as $\delta_i = \sqrt[2]{\lambda_i}$
   2) Form a diagonal matrix S according to the matrix form
   3) Find the set of eigenvectors from the A^TA matrix. Let $\{v_1, v_2, .., v_n\}$ be the eigenvectors of the A^TA matrix with v_1 some of the eigenvectors corresponding to the value of λ_i .
   4) Form an orthogonal matrix V with $V = \{v_1, v_2, .., v_n\}$.
   5) Form a vector set is $\{u_1, u_2, .., u_n\}$ formed for each $1 \le i \le n$ with $u_i = \frac{1}{\delta_i} A v_i$.
   6) Form an orthogonal matrix with $U = \{u_1, u_2, .., u_n\}$.
   7) Use $A=USV^T$ to form SVD decomposition.

This process is called singular value decomposition. The values of S are called singular values of A, the columns of U are the left singular vectors of A and the columns of V are called the right singular vectors of A.

$$S = \begin{bmatrix} \delta_1 & \dots & 0 \\ .. & .. & .. \\ 0 & \dots & \delta_n \end{bmatrix}$$

An example of the SVD application process of document watermarking is as follows:

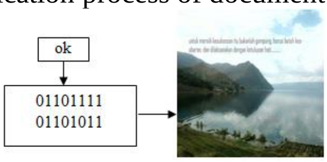

Figure 1. Insertion of Data in Documents

Decimal numbers in text form so that they can be read, they are converted into binary numbers by multiplying each number by two (2) to the power of the number, as below :
1101111= (1x64)+(1x32)+(0x16)+(1x8)+(1x6)+(1x4)+(1x2)+(1x1) = 64 + 32 + 8 + 6 + 4 + 2 +1 = 1111101011 = (1x64)+(1x32)+(0x16)+(1x8)+(0x6)+(1x4)+(1x2)+(1+1) = 64 + 32 + 8 + 4 + 2 + 1 = 107 Then the results of the binary to decimal conversion and seen in the ASCII code. 1101111=111→0 ; 1101011 = 107→k

3.2 SVD Implementation With Watermark

Image quality improvement activities increase ku For an image represented by a matrix $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$, watermark image with matrix $W = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and alpha value = 1.
In general, the singular value decomposition algorithm is as follows : Input : matrix A, Output : orthogonal matrix U, V and the singular matrix S so that $A = USV^T$
The steps in the insertion and extraction process are as follows :

1. $A^TA = \begin{bmatrix} 3 & 8 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} = \begin{bmatrix} 9+64 & 24+-8 \\ 24+0 & 64+1 \end{bmatrix} = \begin{bmatrix} 73 & 16 \\ 24 & 65 \end{bmatrix}$; (73λ) (65λ) – 384 = 0 ;
   $4745 - 75\lambda - 65\lambda + \lambda^2 - 384 = 0$ $\lambda^2 - 138\lambda + 4361 = 0$; $\lambda_1 = 49$; $\lambda_2 = 89$
   calculate the eigenvalues to get the singular value with the determinant matrix A^TA

$A^TA - \lambda = \begin{bmatrix}73-\lambda & 16\\ 24 & 65-\lambda\end{bmatrix}$; $A^TA - \lambda = 0$; $(73-\lambda)(65-\lambda) - 384 = 0$; $4745 - 75\lambda - 65\lambda + \lambda^2 - 384 = 0$;
$\lambda^2 - 138\lambda + 436 = 0$
$(\lambda-9)(\lambda-4) = 0$; $\lambda_1 = 49$; $\lambda_2 = 89$ → nilai eigen

2. Singular value → $\sigma_1 = \sqrt{49} = 7$ dan $\sigma_2 = 89 = 9{,}4339$, so matrix $S = \begin{bmatrix}7 & 0\\ 0 & 9{,}4334\end{bmatrix}$; the form of λ is
$A^TA - \lambda = \begin{bmatrix}73-\lambda & 16\\ 24 & 65-\lambda\end{bmatrix}\begin{bmatrix}x1\\ x2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$; $\lambda - A^TA = \begin{bmatrix}\lambda-73 & -16\\ -24 & \lambda-65\end{bmatrix}\begin{bmatrix}x1\\ x2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$
for $\lambda = 49$ obtained SPL
$= \begin{bmatrix}49-73 & -16\\ -23 & 49-65\end{bmatrix}\begin{bmatrix}x1\\ x2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} = \begin{bmatrix}-24.x1 & -6.x2\\ -24.x1 & -16.x2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} = \begin{bmatrix}-24.x1 + & -6.x2\\ -24.x1 + & -16.x2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$
$\left.\begin{matrix}24.x1 + -6.x2 & = 0\\ 24.x1 + 16.x2 & = 0\end{matrix}\right\}$; $24\,x_1 = -6\,x^2$; $x_1 = \frac{-6x2}{24}$; for $x_2 = S = \begin{bmatrix}x_1\\ x_2\end{bmatrix}$ $x_1 = \frac{-6\,S}{24}$; $x_1 = \frac{-S}{S}$
; for $S = 1 = \begin{bmatrix}-1\\ 4\end{bmatrix}$ Eigenvector $\lambda = 49$; $\lambda = 89$ obtained SPL
$= \begin{bmatrix}89-73 & -6\\ -24 & -89-65\end{bmatrix}\begin{bmatrix}x1\\ x2\end{bmatrix}\begin{bmatrix}0\\ 0\end{bmatrix} = \begin{bmatrix}16 & -6\\ -24 & 24\end{bmatrix}\begin{bmatrix}x1\\ x2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$ $= \left.\begin{matrix}16.x1 - 6.x2 & = 0\\ -24.x1 + 24.x2 & = 0\end{matrix}\right\}$ $16.\,x_1 = 6x_2$; $x_1 = \frac{6x2}{16}$; $x_1 = \frac{3x2}{8}$
for $x_2 = S = \begin{bmatrix}x1\\ x2\end{bmatrix}$ $x_1 = \frac{35}{8} = \frac{3}{8}$; for $S = 1$; $x_1 = \frac{35}{8} = \frac{3}{8}$; $x_2 = S$; $x_2 = 1 = \begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}\frac{3}{8}\\ 1\end{bmatrix}$
then with the results of x1, and x2, the matrix value of S is obtained.

3. Eigenvalues $\lambda_1 = 49$, $\lambda_2 = 89$ each corresponds to an eigenvector
$v_1 = \begin{bmatrix}\frac{-1}{4}\\ 1\end{bmatrix}$ dan $v_2 = \begin{bmatrix}\frac{3}{8}\\ 1\end{bmatrix}$. where is the set of eigenvectors of the matrix A^TA. Suppose $\{v_1, v_2, v_3\}$ are eigenvectors corresponding to the value of λ_i.

4. The set of eigenvectors is autonormal so that a unitary matrix can be formed
$V = [\,v_1\ v_2\,] = \begin{bmatrix}\frac{-1}{4} & \frac{3}{8}\\ 1 & 1\end{bmatrix}$, where V is an orthogonal matrix

5. Then the set of vector matrix U is formed from $u_i = \frac{1}{\sigma_i}Av_i$ so we get :
$U_1 = \frac{1}{\sigma_1}Av_i = \frac{1}{7}\begin{bmatrix}3 & 0\\ 8 & -1\end{bmatrix}\begin{bmatrix}\frac{-1}{4}\\ 1\end{bmatrix} = \frac{1}{7}\begin{bmatrix}\left(3.\frac{-1}{4}\right) + & (0.1)\\ (8.\frac{-1}{4}) & (-1.1)\end{bmatrix} = \frac{1}{7}\begin{bmatrix}\frac{-3}{4}\\ -3\end{bmatrix} = \begin{bmatrix}\frac{-3}{28}\\ \frac{-3}{7}\end{bmatrix}$; $u_2 = \frac{1}{\sigma_i}Av_2 =$
$\frac{1}{9{,}4339}\begin{bmatrix}3 & 0\\ 8 & -2\end{bmatrix}\begin{bmatrix}\frac{3}{8}\\ 1\end{bmatrix} = \frac{1}{9{,}4339}\begin{bmatrix}\frac{9}{8} + & 0\\ 3 + & 1\end{bmatrix} = \frac{1}{9{,}4339}\begin{bmatrix}\frac{9}{8}\\ 4\end{bmatrix} = \begin{bmatrix}\frac{9}{75{,}4712}\\ \frac{4}{9{,}4339}\end{bmatrix}$, After the value of u_1, dan u_2 the value of the vector is formed.

6. We get the orthogonal matrix $U = [u_1 u_2] = \begin{bmatrix}\frac{-3}{28} & \frac{9}{75{,}4712}\\ \frac{-3}{7} & \frac{4}{9{,}4339}\end{bmatrix}$

7. Then we can see the form of SVD (*Singular Value Decomposition*)
$A = USV^T = \begin{bmatrix}\left(\frac{-3}{28}\right) & \frac{9}{75{,}4712}\\ \left(\frac{-3}{7}\right) & \frac{4}{9{,}4339}\end{bmatrix}\begin{bmatrix}7 & 0\\ 0 & 9{,}4339\end{bmatrix}\begin{bmatrix}\frac{-1}{4} & 1\\ \frac{3}{8} & 1\end{bmatrix} = \begin{bmatrix}\frac{-3}{4} & \frac{9}{8}\\ -3 & 4\end{bmatrix}\begin{bmatrix}\frac{-1}{4} & 1\\ \frac{3}{8} & 1\end{bmatrix}$; $A = USV^T =$
$\begin{bmatrix}\frac{81}{16} & \frac{-9}{4}\\ \frac{9}{4} & 1\end{bmatrix}$

8. S_t = S + *alpha* value * $W = \begin{bmatrix} 7 & 0 \\ 0 & 9{,}3439 \end{bmatrix} + 1 \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 8+0 & 7+0 \\ 1+9{,}4339 & 0 \end{bmatrix}$; St = $\begin{bmatrix} 9 & 8 \\ 10{,}4339 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 1 & 10{,}4339 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
$= \begin{bmatrix} 9 & 8 \\ 11{,}4339 & 1 \end{bmatrix}$. Decompose on $S_t = \begin{bmatrix} 7 & 7 \\ 9{,}4339 & 0 \end{bmatrix}$ to get the singular value of S_t. $S_t^T S_t$ = $\begin{bmatrix} 7 & 9{,}4339 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 7 \\ 9{,}4339 & 0 \end{bmatrix} = \begin{bmatrix} 49+88{,}9984 & 49+0 \\ 49+0 & 49+0 \end{bmatrix} = \begin{bmatrix} 137{,}9984 & 49 \\ 49 & 49 \end{bmatrix}$, the value of alpha is the intensity factor that determines the strength of the watermark to be inserted.
9. Calculating the eigenvalues to get the singular value with the determinant of the matrix $S_t^T S_t$
$S_t^T S_t - \lambda = \begin{bmatrix} -137{,}9984-\lambda & 49 \\ 49 & 49-\lambda \end{bmatrix}$; $|St^T \ st| = 0$ (137,9984- λ) (49 - λ) – 2401 = 0
6761,9216 – 137,9984λ - 49λ - λ^2 - 2401 = 0 ; λ^2 – 137,9984λ - 49λ+ 4960,9216 = 0
λ^2 – 186,9984λ + 4960,9216 = 0 ; (λ -37,8173) – (λ-131,181) ; so, λ_1 = 37,8173 dan λ_2=131,181→eigenvalue
Singular value→ $\sigma_1 = \sqrt{37{,}8173}$ = 6,1495 and $\sigma_2 = \sqrt{131{,}181}$ = 11,4534 so matrix S_w = $\begin{bmatrix} 6{,}1995 & 0 \\ 0 & 11{,}4534 \end{bmatrix}$ eigenvalues λ_1 = 6,1995, λ_2 = 11,4534 for λ = 37,8173 =
$\begin{bmatrix} -137{,}9984-\lambda & 49 \\ 49 & 49 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda+137{,}9984 & -49 \\ -49 & -49 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} =$
$\begin{bmatrix} 37{,}8173+137{,}9984 & -49 \\ -49 & -49 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 175{,}8157 & -49 \\ -49 & -49 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} =$
$\begin{bmatrix} 175{,}8157.x1 & -49.x2 \\ -49.x1 & -49.x2 \end{bmatrix} = 0$; 175,8157 x_1 – 49 x_2 = 0175,8157x_1 = 49 x_2 ; $x_1 = \frac{49\ x2}{175{,}8157}$= 0,2787 x_2
for x_2 = S ; x_1 = 0,2787S for x_2 = 1 ; x_1 = 0,2787 . x_2 $\begin{bmatrix} 0{,}2787 \\ 1 \end{bmatrix}$; x_2 = 1 ; for λ = 11,4534
$= \begin{bmatrix} -137{,}9984-\lambda & 49 \\ 49 & 49-\lambda \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda+137{,}9984 & -49 \\ -49 & -49 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} =$
$\begin{bmatrix} 131{,}181+137{,}9984 & -49 \\ -49 & -49 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$= \begin{bmatrix} 269{,}1794 & -49 \\ -49 & -49 \end{bmatrix} = \begin{bmatrix} 269{,}1794.x1 & -49.x2 \\ -49.x1 & -49.x2 \end{bmatrix}$ = 0269,1794 x_1-49x_2 = 0269,1794 x_1 = 49x_2 ; $x_1 = \frac{49x2}{269{,}1794}$= 0,1820 ; x_2
for x_1 = 1 ; x_2 = 0,1820 ; Each corresponds to an eigenvector $v_1 = \begin{bmatrix} 0{,}2787 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0{,}1820 \end{bmatrix}$. vector set
The eigenvectors are autonormal so that a unitary matrix can be formed
$V_w = [v_1 v_2] = \begin{bmatrix} 0{,}2787 & 1 \\ 1 & 0{,}1820 \end{bmatrix}$. Then matrix $U_i = \frac{1}{\sigma_1} S_t V_1 = \frac{1}{6{,}1495} \begin{bmatrix} 7 & 7 \\ 9{,}4339 & 0 \end{bmatrix} \begin{bmatrix} 0{,}2781 \\ 1 \end{bmatrix}$
$= \begin{bmatrix} \frac{7}{6{,}1495} & \frac{7}{6{,}1495} \\ \frac{9{,}4393}{6{,}1495} & \frac{0}{6{,}1495} \end{bmatrix} \begin{bmatrix} 0{,}2781 \\ 1 \end{bmatrix}$= 0,3165 + 1,13830,4266 + 0 = $\begin{bmatrix} 1{,}4548 \\ 0{,}4266 \end{bmatrix}$; $U_2 = \frac{1}{\sigma_1} S_t V_2$ = $\frac{1}{11{,}4534} \begin{bmatrix} 7 & 7 \\ 9{,}4339 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0{,}1820 \end{bmatrix}$
$= \begin{bmatrix} \frac{7}{11{,}4534} & \frac{7}{11{,}4534} \\ \frac{9{,}4339}{11{,}4534} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0{,}1820 \end{bmatrix} = \begin{bmatrix} 1{,}4548 & 0{,}7223 \\ 0{,}4266 & 0{,}8236 \end{bmatrix} = \begin{bmatrix} 0{,}7223 \\ 0{,}8236 \end{bmatrix}$ so matrix
U_w = [u_1 u_2] = $\begin{bmatrix} 1{,}4548 & 0{,}7223 \\ 0{,}4266 & 0{,}8236 \end{bmatrix}$. ; $S_t = U_w S_w V_W^T$ = $\begin{bmatrix} 1{,}4548 & 0{,}7223 \\ 0{,}4266 & 0{,}8236 \end{bmatrix} \begin{bmatrix} 6{,}1995 & 0 \\ 0 & 11{,}4534 \end{bmatrix} \begin{bmatrix} 0{,}2787 & 1 \\ 1 & 0{,}1820 \end{bmatrix}$.
$= \begin{bmatrix} 9{,}0190 & 8{,}2727 \\ 0{,}7370 & 9{,}4330 \end{bmatrix} \begin{bmatrix} 0{,}2787 & 1 \\ 1 & 0{,}1820 \end{bmatrix} = \begin{bmatrix} 10{,}7862 & 10{,}5246 \\ 9{,}6384 & 2{,}4538 \end{bmatrix}$

10. $A_w = US_wV^T$; $A_w = \begin{bmatrix} \frac{-3}{28} & \frac{9}{75{,}4712} \\ \frac{-3}{7} & \frac{4}{9{,}4339} \end{bmatrix} \begin{bmatrix} 6{,}1995 & 0 \\ 0 & 11{,}4534 \end{bmatrix} \begin{bmatrix} \frac{-1}{4} & 1 \\ \frac{3}{8} & 1 \end{bmatrix}$.=

$\begin{bmatrix} -0{,}6639 & 1{,}3652 \\ 2{,}6564 & 4{,}8562 \end{bmatrix} \begin{bmatrix} \frac{-1}{4} & 1 \\ \frac{3}{8} & 1 \end{bmatrix}$;

$A_w = \begin{bmatrix} 1{,}1898 & 2{,}0291 \\ 2{,}4851 & 7{,}5126 \end{bmatrix} \rightarrow$ watermarked image

The result of Aw is the last step of embedding, and then the obtained Sw is used together with the U and V matrices of the initial image to form a watermarked image.

11. The value of the extraction process can be obtained by W=$\frac{S_{t-S}}{alpha\ values}$

$W = \frac{S_{t-S}}{alpha\ values} = \frac{\begin{vmatrix} \frac{9}{11{,}4339} & \frac{8}{1} \end{vmatrix} - \begin{bmatrix} \frac{8}{10{,}4339} & \frac{7}{1} \end{bmatrix}}{1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

In the extraction process, the decomposition of the watermark image matrix Aw Thus the value of the singular value Sw is obtained, and the matrix St is obtained by using the obtained singular value Sw, which is U_w and $v_W^T \rightarrow U_wS_wv_W^T$, then the watermark image is obtained from the value in the extraction process above.

4. Conclusion

Based on the results of the discussion, the following conclusions are drawn The watermarking technique has the principle of increasing the level of document security in addition to providing information about the ownership authority to be more secure, Application of watermarking with the Singular Value Decomposition method, which is one of the techniques in numerical analysis that to diagonalize a matrix from the point of view of the value of the image matrix to produce orthogonal values of U, V and singular matrix S so that $A = US_wV^T$ can vary, The design of the watermark application can help users to attach a watermark to the document. In addition to increasing the artistic value, it also helps to ensure the ownership of the document.

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